Published by Institute of Physics Publishing for SISSA/ISAS

Received: *January 24, 2003*
Revised: *March 20, 2003*
Accepted: *April 8, 2003*

# Higher spins from tensorial charges and $\mathrm{OSp}(N|2n)$ symmetry

**Mikhail Plyushchay**

*Departamento de Física, Universidad de Santiago de Chile*
*Casilla 307, Santiago 2, Chile, and*
*Institute for High Energy Physics, Protvino, Russia*
*E-mail:* mplyushc@lauca.usach.cl

**Dmitri Sorokin**

*Institute for Theoretical Physics, NSC KIPT*
*61108 Kharkov, Ukraine, and*
*Dipartimento di Fisica, Università degli Studi di Padova, and Istituto Nazionale di Fisica Nucleare, Sezione di Padova, Via F. Marzolo 8, 35131 Padova, Italy*
*E-mail:* dmitri.sorokin@pd.infn.it

**Mirian Tsulaia**

*Dipartimento di Fisica, Università degli Studi di Padova, and Istituto Nazionale di Fisica Nucleare, Sezione di Padova*
*Via F. Marzolo 8, 35131 Padova, Italy, and*
*Bogoliubov Laboratory of Theoretical Physics, JINR*
*141980 Dubna, Russia, and*
*Institute of Physics, GAS, 380077 Tbilisi, Georgia*
*E-mail:* mirian.tsulaia@pd.infn.it

Abstract: It is shown that the quantization of a superparticle propagating in an $N=1$, $D=4$ superspace extended with tensorial coordinates results in an infinite tower of massless spin states satisfying the Vasiliev unfolded equations for free higher spin fields in flat and $AdS_4$ $N=1$ superspace. The tensorial extension of the $AdS_4$ superspace is proved to be a supergroup manifold $\mathrm{OSp}(1|4)$. The model is manifestly invariant under $\mathrm{OSp}(N|8)$ ($N=1,2$) superconformal symmetry. As a byproduct, we find that the Cartan forms of arbitrary $\mathrm{Sp}(2n)$ and $\mathrm{OSp}(1|2n)$ groups are $\mathrm{GL}(2n)$ flat, i.e. they are equivalent to flat Cartan forms up to an exactly determined $\mathrm{GL}(2n)$ rotation. This property is crucial for carrying out the quantization of the particle model on $\mathrm{OSp}(1|4)$ and getting the higher spin field dynamics in super $AdS_4$, which can be performed in a way analogous to the flat case.

Keywords: Conformal Field Models in String Theory, AdS-CFT and dS-CFT Correspondence, Space-Time Symmetries, Superspaces.





JHEP04(2003)013

## Contents



## 1. Introduction

It is well known that the higher spin fields naturally arise in the quantization of classical superstrings and describe an infinite tower of quantum string states with masses linearly depending on spin and on inverse string tension. In the limit where the string tension tends to zero all (higher spin) string states become massless and may be regarded as gauge fields of an infinite, so called, higher spin symmetry. There is a conjecture that string theory is a spontaneously broken phase of a gauge theory of higher spin fields. If this conjecture is realized, it can be useful for better understanding of string/M theory and of the AdS/CFT correspondence (see e.g. [1]–[3]). This is one of the motivations of the development of the theory of interacting higher spin fields.

Understanding the interactions of higher spin fields is a main problem of the construction of the higher spin field theory, the gauge invariance playing a crucial role in cancelling the contribution of the states of negative norm. The interaction problem already reveals itself when one tries to couple higher spin fields to gravity. As it has been realized in [4] a space-time with a constant curvature, in particular an Anti de Sitter space, rather than a flat space, is an appropriate vacuum background on which the construction of a consistent interacting theory of higher spin fields should be based.

There are at least two alternative approaches to the description of massless higher spin fields both in flat and in AdS backgrounds (see e.g. [5] for a recent review and for the



JHEP04(2003)013

construction of conformal higher spin theory). One of the approaches is the formulation in terms of totally symmetric tensor fields (see [6] for the case of $AdS_4$, [7, 8] for the case of arbitrary dimensional AdS spaces and [9] for the latest developments). Another approach is a geometrical formulation [4, 1], [10]–[15] in which an important role in the construction of interacting higher spin field theory is played by the algebra of higher spin symmetries $hs(2n)$. Since the number of higher spin fields is infinite the algebra $hs(2n)$ is infinite dimensional and contains the algebra of isometries of the AdS space as a finite dimensional subalgebra. The realization of the higher spin algebra on the higher spin fields is described with the help of auxiliary spinor variables which are very similar to twistors.

This fact may have quite interesting implications. One may ask if there exists a (super)particle model whose quantization would produce an infinite set of the higher spin fields and whether it may be used to understand how the higher spin interactions can be introduced. Since together with an AdS background, the consistent interactions of the higher spin fields require that the whole infinite tower of them be involved, the quantum states of such a superparticle should form the infinite higher spin set, and not a finite one, as conventional superparticle and spinning particle models have. In other words, the theory should not have the constraint which fixes the value of the second Casimir operator (helicity). Twistors turn out to be rather useful to construct such a model. In [16] there has been proposed a model of a massless superparticle in an $N = 1$, $D = 4$ superspace enlarged with tensorial coordinates and twistor-like spinor variables. The conjugate momenta of these tensorial coordinates appear as tensorial charges in a superalgebra which generates the supersymmetry of the system. It is worth noting that this model has been the first example of BPS states which preserve more than one half supersymmetry. (A generalization of this model to a massive superparticle has been discussed in [17]. See also [18, 19] for further developments and applications to the case of branes). The quantum spectrum of the massless tensorial superparticle has been shown [20] to consist of an infinite tower of massless higher (integer and half integer) spin states, while the twistorial variables become noncommutative, and it has been assumed that this theory corresponds to the higher spin field theory developed by Vasiliev [10]–[12].

The aim of the present paper is to study in more detail this relationship and, in particular, to analyze the OSp(1|8) and OSp(2|8) superconformal invariance of the model and to derive the so called 'unfolded' equations of motion of higher spin fields [10]–[12] which appear as a result of the quantization of the superparticle.

We start with the superparticle model in the flat $N = 1$, $D = 4$ superspace extended with tensorial coordinates and determine its OSp($N$|8) ($N = 1, 2$) structure. We then pass to a superparticle in an $N = 1$ supersymmetric $AdS_4$ background and show that its tensorial extension is a supergroup manifold OSp(1|4). The classical dynamics of a superparticle on super $AdS_4$ and on OSp(1|4) has already been considered in [21].

At this point we should note that since the relationship of the quantized superparticle models of [20, 21] to the unfolded higher spin field equations was not clear, in [11] the actions for these superparticles were enlarged with new variables along with kinetic terms for twistorial coordinates. Such a modification changed the structure of first and second class constraints but at the same time kept the number of physical degrees of freedom



JHEP04(2003)013

of the models intact. This generalization is similar to and is a lagrangian form of the conversion procedure used in [20] to quantize the superparticle by converting second class constraints into first class constraints with the help of new auxiliary variables. In [11] this allowed to carry out a BRST quantization of these generalized superparticles which yielded unfolded equations of motion of higher spin fields as functions of the coordinates of the generalized tensorial superspace. The same equations, in a momentum representation for spinorial variables, were obtained in [20], but their meaning as unfolded equations was not understood therein. A main result of [11] has been that using arguments by Fronsdal [22], who was actually the first to realize the importance of simplectic group manifolds for the description of higher spins, it was shown that in $D = 4$ the dynamics in the tensorial space is locally and globally equivalent to the unfolded free higher spin field dynamics in flat Minkowski and $AdS$ spaces.

In the present paper we show that it is possible to avoid the introduction of new variables and of the corresponding kinetic terms. The superparticle models [20, 21] are quantized in a (twistor) momentum representation in contrast to the tensorial space coordinate quantization performed in [11]. This allows us, upon applying an appropriate Fourier transform, to get higher spin wave functions propagating and satisfying the unfolded equations directly in the physical Minkowski and $AdS$ subspaces of the tensorial superspace.

We also demonstrate that both, the flat tensorial superspace and OSp(1|4) can be regarded as different coset superspaces of the form $\frac{\mathrm{OSp}(1|8)}{\mathrm{GL}(4)\otimes SK}$, where $\mathrm{GL}(4)\otimes SK$ is a semidirect product of the general linear group GL(4) and of a generalized super Poincare group $SK$, which are subgroups of OSp(1|8). As a consequence the Cartan forms (or supervielbeins) of the two supermanifolds are related to each other by a GL(4) transformation. As a byproduct we find that the Cartan one-forms of any supergroup $\mathrm{OSp}(1|2n)$ $(n = 1, \ldots, \infty)$ are $\mathrm{GL}(2n)$ flat, i.e. that they differ from the Cartan forms of a corresponding flat tensorial superspace by a definite explicitly found $\mathrm{GL}(2n)$ rotation (see section 6). Such peculiar properties of $\mathrm{OSp}(1|2n)$ and in particular of the OSp(1|4) supermanifold will allow us to quantize the superparticle on OSp(1|4) in a simple way by applying the twistor technique used for the quantization of the superparticle in flat tensorial superspace [20] and adopted below to the derivation of the unfolded higher spin field equations in Minkowski and $AdS_4$ superspaces.

Our conclusion, which is in agreement with the philosophy of [11, 12] and [20]–[22], is that tensorial spaces, and in particular supergroup manifolds $\mathrm{OSp}(N|2n)$, should play more fundamental role in the construction of the higher spin field theory than the conventional Minkowski or AdS space-time.

## 2. Review of the tensorial particle model

The action of the $N = 1$, $D = 4$ superparticle with tensorial coordinates has the following form

$$S = \int d\tau \; \lambda_\alpha \lambda_\beta \, (\partial_\tau X^{\alpha\beta} - i \, \partial_\tau \theta^\alpha \, \theta^\beta) \,, \tag{2.1}$$

where

$$X^{\alpha\beta} = X^{\beta\alpha} = \frac{1}{2}\gamma^{\alpha\beta}_m\, x^m + \frac{1}{4}\gamma^{\alpha\beta}_{mn}\, y^{mn}\,, \tag{2.2}$$

$x^m = \gamma^m_{\alpha\beta}\, X^{\alpha\beta}$ are conventional $D = 4$ space-time coordinates ($m$=0,1,2,3), $y^{mn} = \gamma^{mn}_{\alpha\beta} X^{\alpha\beta}$ are antisymmetric tensor coordinates, $\theta^\alpha$ are anticommuting Majorana spinor coordinates and $\lambda_\alpha$ are auxiliary commuting Majorana spinor variables ($\alpha = 1, \ldots, 4$). The physical meaning of the tensorial coordinates $y^{mn}$ is to provide the superparticle, upon qunatization, with infinitely many integer and half-integer spinning degrees of freedom [20]. So the generalized superspace ($X^{\alpha\beta}$, $\theta^\alpha$) can be regarded as an extended 'higher-spin' superspace which, as we shall see below, possesses an OSp(1|8) symmetry (see also [11, 12] and references therein) and can be realized as a coset superspace $\frac{\mathrm{OSp}(1|8)}{\mathrm{GL}(4)\otimes SK}$, where GL(4) $\otimes$ $SK$ is a semidirect product of the general linear group GL(4) and a generalized super Poincare group $SK$ which are subgroups of OSp(1|8).

The action is manifestly invariant under SO(1, 3) Lorentz rotations

$$\delta\lambda_\alpha = l_\alpha{}^\beta\lambda_\beta\,, \qquad \delta\theta_\alpha = l_\alpha{}^\beta\theta_\beta\,, \qquad \delta X^{\alpha\beta} = -X^{\alpha\gamma}\, l_\gamma{}^\beta - X^{\gamma\beta}\, l_\gamma{}^\alpha\,, \tag{2.3}$$

constant translations in the space of $X^{\alpha\beta}$

$$\delta X^{\alpha\beta} = a^{\alpha\beta} \tag{2.4}$$

and under global $N = 1$, $D = 4$ supersymmetry transformations

$$\delta\theta^\alpha = \epsilon^\alpha, \quad \delta X^{\alpha\beta} = -\frac{i}{2}(\epsilon^\alpha\theta^\beta + \epsilon^\beta\theta^\alpha)\,, \tag{2.5}$$

the corresponding superalgebra being

$$\{Q_\alpha, Q_\beta\} = P_{\alpha\beta} = -2\gamma^m_{\alpha\beta}\, P_m + \gamma^{mn}_{\alpha\beta}\, Z_{mn}\,, \qquad [P_{\alpha\beta}, P_{\gamma\delta}] = [P_{\alpha\beta}, Q_\gamma] = 0\,, \tag{2.6}$$

where $Q_\alpha$ are supersymmetry generators, $P_m$ are $D = 4$ translations and $Z_{mn} = -Z_{nm}$ are tensorial charges generating the translations along $y^{mn}$.

From eq. (2.1) it follows that the particle momentum in the $D = 4$ space is

$$P_m = \frac{1}{2}\lambda\gamma_m\lambda \quad \Rightarrow \quad P_m P^m = 0\,, \tag{2.7}$$

and hence the particle is massless. The momenta conjugate to $y^{mn}$ are also constrained to be the bilinear combinations of the commuting spinors

$$Z_{mn} = \frac{1}{4}\lambda\gamma_{mn}\lambda\,. \tag{2.8}$$

The constraints (2.7), (2.8) together with the constraints on the momenta of $\lambda$

$$P^\alpha = 0\,, \tag{2.9}$$

and constraints on the momenta conjugate to $\theta$

$$\pi_\alpha = \lambda_\alpha\,\lambda_\beta\,\theta^\beta \tag{2.10}$$

imply that in the phase space the superparticle (2.1) has only eight bosonic and one fermionic degrees of freedom (see [20] for the details).

The constraints (2.7)–(2.10) are a mixture of the first and second class constraints. As has been mentioned in the Introduction, one of the ways to handle these constraints and to perform quantization considered in [20] is to convert all the constraints into the first class by introducing auxiliary variables. This can be done [11] by adding to the action (2.1) the terms of the form

$$S_{\text{add}} = \int d\tau [\lambda_\alpha \partial_\tau P^\alpha - i\chi(\partial_\tau \chi - \lambda_\alpha \partial_\tau \theta^\alpha)]\,, \tag{2.11}$$

where $\chi(\tau)$ is a Grassmann odd variable and $\lambda_\alpha$ is regarded as a momentum conjugate to $P^\alpha$ which now is unconstrained.

In what follows we shall follow an alternative way. The constraints can be implicitly solved in terms of independent $(8_b, 1_f)$ supertwistor degrees of freedom by rewriting the action (2.1) in the following supertwistor form

$$S = -\int d\tau\ \mathcal{Z}^a\, \partial_\tau \mathcal{Z}_a\,, \tag{2.12}$$

where

$$\mathcal{Z}_a = (\lambda_\alpha,\, \mu^\beta,\, \chi) \tag{2.13}$$

is the supertwistor and

$$\mu^\alpha = \left(X^{\alpha\beta} - \frac{i}{2}\theta^\alpha\,\theta^\beta\right)\lambda_\beta\,, \qquad \chi = \theta^\alpha \lambda_\alpha \tag{2.14}$$

(small Latin letters from the beginning of the alphabet stand for the supertwistor indices).

The supertwistor index is raised and lowered by the OSp(1|8) invariant matrix

$$C^{ab} = \begin{pmatrix} 0 & \delta_\beta^\alpha & 0 \\ -\delta_\alpha^\beta & 0 & 0 \\ 0 & 0 & -i \end{pmatrix}. \tag{2.15}$$

$\mathcal{Z}^a$ is the "conjugate" supertwistor

$$\mathcal{Z}^a = C^{ab}\mathcal{Z}_b = (\mu^\alpha,\, -\lambda_\beta,\, -i\chi) \tag{2.16}$$

so that the bilinear form $\mathcal{Z}_1^a \mathcal{Z}_{2a}$ is OSp(1|8) invariant and so does the action (2.12). This implies that also the action (2.1) possesses the OSp(1|8) symmetry, and we should only find corresponding OSp(1|8) variations of the coordinates $X^{\alpha\beta}$ and $\theta^\alpha$.

## 3. The OSp(1|8) transformations

The supertwistors (2.13) and (2.16) transform under a fundamental linear representation of OSp(1|8). The infinitesimal OSp(1|8) transformations

$$\delta \mathcal{Z}^a = \Xi^{ab}\mathcal{Z}_b \tag{3.1}$$

are given by symmetric matrices

$$\Xi^{ab} = \begin{pmatrix} a^{\alpha\beta} & g_\beta^{\ \alpha} & -i\epsilon^\alpha \\ g_\alpha^{\ \beta} & k_{\alpha\beta} & -i\kappa_\alpha \\ -i\epsilon^\beta & -i\kappa_\beta & 0 \end{pmatrix}. \tag{3.2}$$

The corresponding symmetric matrix of the OSp(1|8) generators is

$$T_{ab} = \begin{pmatrix} P_{\alpha\beta} & G_\alpha{}^\beta & Q_\alpha \\ G_\beta{}^\alpha & K^{\alpha\beta} & S^\alpha \\ Q_\beta & S^\beta & 0 \end{pmatrix} . \tag{3.3}$$

In terms of the canonical supertwistor variables $\mathcal{Z}_a$ (2.13), (2.14) satisfying the Poisson bracket relations $[\mathcal{Z}_a,\, \mathcal{Z}_b] = C_{ab}$, the OSp(1|8) generators are realized as follows

$$T_{ab} = \mathcal{Z}_a \mathcal{Z}_b = \begin{pmatrix} \lambda_\alpha \lambda_\beta & \lambda_\alpha \mu^\beta & \lambda_\alpha \chi \\ \mu^\alpha \lambda_\beta & \mu^\alpha \mu^\beta & \mu^\alpha \chi \\ \lambda_\beta \chi & \mu^\beta \chi & 0 \end{pmatrix} . \tag{3.4}$$

The components of (3.3) satisfy the $osp(1|8)$ superalgebra

$$[P,P] = 0\,, \qquad [K,K] = 0\,, \tag{3.5}$$

$$[P,K] \sim G\,, \qquad [G,G] \sim G\,, \qquad [G,P] \sim P\,, \qquad [G,K] \sim K\,, \tag{3.6}$$

$$[G,Q] \sim Q\,, \qquad [G,S] \sim S\,, \qquad \{Q,Q\} \sim P\,, \qquad \{S,S\} \sim K\,, \qquad \{S,Q\} \sim G\,, \tag{3.7}$$

where

$$T_{\hat\alpha\hat\beta} = \begin{pmatrix} P_{\alpha\beta} & G_\alpha{}^\beta \\ G_\beta{}^\alpha & K^{\alpha\beta} \end{pmatrix}, \qquad (\hat\alpha, \hat\beta = 1, \ldots, 8) \tag{3.8}$$

are the generators of the Sp(8) subgroup of OSp(1|8).

The components of $T_{\hat\alpha\hat\beta}$ are the GL(4) general linear transformations $G_\alpha{}^\beta$ , the symmetric commuting tensorial charges $P_{\alpha\beta}$ which include the $D = 4$ translations $P_m$ and the tensorial boosts $Z_{mn}$, and which together with the fermionic charges $Q_\alpha$ generate the supersymmetry algebra (2.6), and 'generalized conformal' boosts

$$K^{\alpha\beta} = K^{\beta\alpha} = \gamma_m^{\alpha\beta}\, K^m \,+\, \gamma_{mn}^{\alpha\beta}\, K^{mn}\,, \tag{3.9}$$

where $K^m$ are the standard special conformal generators and $K^{mn} = -K^{nm}$ are their tensorial counterparts.

The SO(1, 3) spinor indices can be raised and lowered with antisymmetric charge conjugation matrices $C^{\alpha\beta}$ and $C_{\alpha\beta}$.

$$\lambda^\alpha = C^{\alpha\beta} \lambda_\beta\,, \qquad \mu_\alpha = -C_{\alpha\beta}\, \mu^\beta\,, \qquad C^{\alpha\gamma} C_{\gamma\beta} = -\delta^\alpha_\beta\,. \tag{3.10}$$

Let us consider the matrix $G_{\alpha\beta} = G_\alpha{}^\gamma\, C_{\gamma\beta}$ obtained by lowering the index of the GL(4) matrix. The symmetric part of $G_{\alpha\beta}$ generates an $\mathrm{Sp}(4) \sim \mathrm{SO}(2,3)$ subgroup of Sp(8) which contains the $D = 4$ Lorentz group generated by $L_{mn}$ and vector charge generators $Z_m$ which can be regarded as $AdS_4$ boosts

$$G_{\{\alpha\beta\}} \equiv M_{\alpha\beta} = \gamma^m_{\alpha\beta}\, Z_m \,+\, \gamma^{mn}_{\alpha\beta}\, L_{mn}\,. \tag{3.11}$$

The antisymmetric part of $G_{\alpha\beta}$ contains dilatation $D$, a $U(1)$ generator U and a 3-form charge $Z_{mnp}$

$$G_{[\alpha\beta]} = \frac{1}{2} C_{\alpha\beta}\, D + \gamma^5_{\alpha\beta}\, U + \gamma^{mnp}_{\alpha\beta}\, Z_{mnp}\,. \tag{3.12}$$



JHEP04(2003)013

OSp(1|8) can be regarded as a generalized $N=1$, $D=4$ superconformal group with $S^\alpha$ being superconformal generators

$$\{S^\alpha, S^\beta\} = K^{\alpha\beta}\,. \tag{3.13}$$

Note that $S^\alpha$ and $K^{\alpha\beta}$ generate a second copy of the generalized super Poincare algebra (SK) similar to (2.6).

OSp(1|8) contains the conformal group SO(2,4) as a subgroup generated by $P_m$, $L_{mn}$, $D$ and $K^m$, but the superconformal group SU(2,2|1) is not a subgroup of OSp(1|8) because of the reasons explained in [23]. SU(2,2|1) is a subgroup of a larger simple supergroup OSp(2|8). We shall consider a generalization of the superparticle model (2.1), which is OSp(2|8) invariant, in the next section 4.

Using the variation law (3.1), (3.2) and the particular form of the supertwistor components (2.13)–(2.16) we can easily get the OSp(1|8) variation of $\lambda_\alpha$, $\theta^\alpha$ and $X^{\alpha\beta}$

$$\delta\lambda_\alpha = g_\alpha{}^\beta\lambda_\beta - k_{\alpha\beta}(X^{\beta\gamma} - \frac{i}{2}\theta^\beta\theta^\gamma)\lambda_\gamma - i\kappa_\alpha\theta^\beta\lambda_\beta\,, \tag{3.14}$$

$$\delta\theta^\alpha = \epsilon^\alpha - \theta^\beta\, g_\beta{}^\alpha + \theta^\beta\, k_{\beta\gamma}X^{\gamma\alpha} - \kappa_\beta\left(X^{\beta\alpha} - \frac{i}{2}\theta^\beta\theta^\alpha\right) - i\theta^\alpha\theta^\beta\kappa_\beta\,, \tag{3.15}$$

$$\begin{aligned}\delta X^{\alpha\beta} &= \left[a^{\alpha\beta} - \frac{i}{2}\left(\epsilon^\alpha\theta^\beta + \epsilon^\beta\theta^\alpha\right)\right] - (X^{\gamma\beta}g_\gamma{}^\alpha + X^{\gamma\alpha}g_\gamma{}^\beta) + X^{\alpha\gamma}k_{\gamma\delta}X^{\delta\beta} - \\ &\quad - \frac{i}{2}\theta^\alpha X^{\beta\gamma}\kappa_\gamma - \frac{i}{2}\theta^\beta X^{\alpha\gamma}\kappa_\gamma\,.\end{aligned} \tag{3.16}$$

In (3.15) and (3.16) the first terms are $N=1$ supersymmetry transformations (2.4), (2.5) in the $D=4$ superspace extended with the tensorial directions, the second terms contain SO(1,3) Lorentz rotations (3.11) (when $g^{\alpha\beta} = l^{mn}\,\gamma_{mn}^{\alpha\beta}$) and dilatations (3.12) (when $g^{\alpha\beta} = \frac{1}{2}C^{\alpha\beta}\,\phi$), and the remaining terms are generalized conformal and superconformal boosts.

The action (2.1), as well as the constraints and the equations of motion which it produces, are invariant under the OSp(1|8) transformations (3.14)–(3.16). Note that under (3.14)–(3.16) the one forms

$$\Pi^{\alpha\beta} = dX^{\alpha\beta} - \frac{i}{2}\left(d\theta^\alpha\,\theta^\beta + d\theta^\beta\,\theta^\alpha\right), \qquad E^\alpha = d\theta^\alpha \tag{3.17}$$

transform as follows

$$\delta\Pi^{\alpha\beta} = \Pi^{\alpha'\beta}\, g_{\alpha'}{}^\alpha(X,\theta) + \Pi^{\alpha\beta'}\, g_{\beta'}{}^\beta(X,\theta)\,, \tag{3.18}$$

where $g_{\alpha'}^{\alpha}(X,\theta)$ are the infinitesimal OSp(1|8) transformations nonlinearly realized on the coset superspace $\frac{\mathrm{OSp}(1|8)}{\mathrm{GL}(4)\otimes SK}$ in terms of GL(4) matrices. $SK$ stands for the copy of the generalized super Poincare subgroup of OSp(1|8) generated by $S_\alpha$ and $K_{\alpha\beta}$ (3.13). We have thus demonstrated that the generalized superspace $(X^{\alpha\beta},\,\theta^\alpha)$ is the coset superspace $\frac{\mathrm{OSp}(1|8)}{\mathrm{GL}(4)\otimes SK}$.

Note that the finite OSp(1|8) variations of $\lambda_\alpha$ and $\Pi^{\alpha\beta}$ are

$$\hat\lambda_\alpha = G^{-1\alpha'}_\alpha(X,\theta)\,\lambda_{\alpha'}\,, \qquad \hat\Pi^{\alpha\beta} = \Pi^{\alpha'\beta'}\,G_{\alpha'}{}^\alpha(X,\theta)\,G_{\beta'}{}^\beta(X,\theta)\,, \tag{3.19}$$

from which one immediately sees that the action (2.1) is OSp(1|8) invariant.



JHEP04(2003)013

## 4. The OSp(2|8) invariant model

As we have mentioned, a conventional superconformal group SU(2, 2|1) is not a subgroup of the supergroup OSp(1|8) but is a subgroup of OSp(2|8). As it has been found in [20], there exists a generalization of the action (2.1) which is OSp(2|8) invariant. It has the following form

$$S=\int d\tau\,\lambda_\alpha\lambda_\beta\left(\partial_\tau X^{\alpha\beta}-\frac{i}{4}\,\partial_\tau\theta\gamma^m\theta\gamma_m^{\alpha\beta}+\frac{i}{8}\,a\,\partial_\tau\theta\gamma^{mn}\theta\gamma_{mn}^{\alpha\beta}\right), \tag{4.1}$$

where $0\le a\le 1$ is a numerical parameter and

$$X^{\alpha\beta}=X^{\beta\alpha}=\frac{1}{2}\gamma_m^{\alpha\beta}\,x^m\;+\;\frac{a}{4}\gamma_{mn}^{\alpha\beta}\,y^{mn}\,. \tag{4.2}$$

When $a=1$ eq. (4.1) reduces to (2.1) due to the Fierz identity

$$C^{\alpha\{\beta}C^{\gamma\}\delta}=\frac{1}{4}\gamma^{m\alpha\delta}\gamma_m^{\beta\gamma}-\frac{1}{8}\gamma^{mn\alpha\delta}\gamma_{mn}^{\beta\gamma}\,. \tag{4.3}$$

Using other Fierz identities the action (4.1) can be written as

$$S=\int d\tau\,\lambda_\alpha\lambda_\beta\left[\partial_\tau X^{\alpha\beta}-\frac{i(1+a)}{2}\,\partial_\tau\theta^\alpha\theta^\beta+\frac{i(1-a)}{2}\,(\partial_\tau\theta\gamma^5)^\alpha(\theta\gamma^5)^\beta\right], \tag{4.4}$$

or in a manifestly invariant OSp(2|8) supertwistor form as

$$S=-\int d\tau\,\mathcal{Z}^{\hat a}\,\partial_\tau\mathcal{Z}_{\hat a}\,, \tag{4.5}$$

where

$$\mathcal{Z}_{\hat a}=(\lambda_\alpha,\,\mu^\beta,\,\chi^i),\qquad i=1,2 \tag{4.6}$$

and

$$\begin{aligned}\mu^\alpha&=\left[X^{\alpha\beta}-\frac{i(1+a)}{4}\,\theta^\alpha\,\theta^\beta-\frac{i(1-a)}{4}\,(\theta\gamma^5)^\alpha\,(\theta\gamma^5)^\beta\right]\lambda_\beta\,,\\ \chi^1&=\sqrt{\frac{1+a}{2}}\,\theta^\alpha\lambda_\alpha\qquad \chi^2=i\sqrt{\frac{1-a}{2}}\,\theta\gamma^5\lambda\,.\end{aligned} \tag{4.7}$$

The supertwistor index is raised and lowered by the OSp(2|8) invariant matrix

$$C^{\hat a\hat b}=\begin{pmatrix}0&\delta_\alpha^\beta&0\\-\delta_\beta^\alpha&0&0\\0&0&-i\delta^{ij}\end{pmatrix}. \tag{4.8}$$

As in the previous section 3 starting from linear OSp(2|8) transformations of the supertwistor (4.6) generated by the matrices

$$T_{\hat a\hat b}=\begin{pmatrix}P_{\alpha\beta}&G_\alpha^{\ \beta}&Q_\alpha^j\\G_\beta^{\ \alpha}&K^{\alpha\beta}&S^{j\alpha}\\Q_\beta^i&S^{i\beta}&\epsilon^{ij}\end{pmatrix}, \tag{4.9}$$

one can find corresponding OSp(2|8) variations of $X^{\alpha\beta}$ and $\theta^\alpha$.

## 5. Quantization and higher spin field equations of motion

The quantization of the superparticle models described by the actions (2.1), (4.1) and (4.4) has been considered in detail in [20] so we shall just present main results which are a direct consequence of the quantization of the free supertwistor models (2.12) and (4.5). In particular, for the values of the parameter $a \neq 0, 1$ the action (4.5) is of the first order form with $\lambda^\alpha$ and $\chi$ being the coordinates and $\mu_\alpha$ and $2i\bar{\chi}$ being the corresponding conjugate momenta. Upon quantization the Poisson brackets of the canonical variables

$$[\mu_\alpha, \lambda^\beta]_P = \delta^\alpha_\beta\,, \qquad \{\bar{\chi}, \chi\}_P = -\frac{i}{2} \tag{5.1}$$

are replaced with the (anti)commutators according to the rule $[\ ,\ ]_P \ \rightarrow\ i[\ ,\ ]$ and $\{\ ,\ \}_P\ \rightarrow -i\{\ ,\ \}$, and the quantum states of the superparticle are described by a wave function of $\lambda_\alpha$ and $\chi = \chi^1 + i\chi^2$

$$\Phi(\lambda_\alpha, \chi) = \sum_{n=0}^{\infty} \lambda_{\alpha_1}\cdots\lambda_{\alpha_n}(\phi^{\alpha_1\cdots\alpha_n} + i\chi\,\psi^{\alpha_1\cdots\alpha_n})\,. \tag{5.2}$$

For further analysis it is useful to rewrite the wave function (5.2) in a two-component Weyl representation of the spinor $\lambda^\alpha$

$$\lambda^\alpha = (\lambda^A, \bar{\lambda}^{\dot{A}})\,, \qquad (A, \dot{A} = 1, 2)\,, \tag{5.3}$$

where $\lambda^A$ and $\bar{\lambda}^{\dot{A}}$ are complex conjugate Weyl spinors. We thus have

$$\begin{aligned}\Phi(\lambda_A, \bar{\lambda}_{\dot{A}}, \chi) = &\sum_{n=0}^{\infty} \lambda_{A_1}\cdots\lambda_{A_n}[\phi^{A_1\cdots A_n}(\lambda_A\bar{\lambda}_{\dot{A}}) + i\chi\,\psi^{A_1\cdots A_n}(\lambda_A\bar{\lambda}_{\dot{A}})] + \\ &+ \sum_{n=0}^{\infty} \bar{\lambda}_{\dot{A}_1}\cdots\bar{\lambda}_{\dot{A}_n}[\bar{\phi}^{\dot{A}_1\cdots\dot{A}_n}(\lambda_A\bar{\lambda}_{\dot{A}}) + i\chi\,\bar{\psi}^{\dot{A}_1\cdots\dot{A}_n}(\lambda_A\bar{\lambda}_{\dot{A}})]\,,\end{aligned} \tag{5.4}$$

where $\phi(\lambda_A\bar{\lambda}_{\dot{A}})$ and $\psi(\lambda_A\bar{\lambda}_{\dot{A}})$ depend on the product of the complex conjugate spinors, which because of (2.7) is equal to the superparticle momentum

$$P_{A\dot{A}} \equiv \sigma^m_{A\dot{A}}P_m = \lambda_A\bar{\lambda}_{\dot{A}}\,, \tag{5.5}$$

$\sigma^m_{A\dot{A}}$ being Pauli matrices. Therefore, the functions (5.2) and (5.4) describe quantum states of the superparticle in the momentum representation. This has been obtained in [20].

In the case $a = 1$ there is only one real fermionic variable $\chi$ (see eq. (4.7)) which becomes a Clifford variable ($(\chi)^2 = 1$) when one applies the standard Dirac quantization procedure and solves the second class constraint relating $\chi$ with its conjugate momentum. As a result the wave functions (5.2) and (5.4) become Clifford 'superfields' [20].

Finally, when $a = 0$ in the action (4.4) there is an additional first class constraint

$$\mu^A\lambda_A - \bar{\mu}^{\dot{A}}\bar{\lambda}_{\dot{A}} + 2i\bar{\chi}\chi = 0\,. \tag{5.6}$$

This constraint implies a helicity condition on the wave function (5.4)

$$\left(\lambda_A\frac{\partial}{\partial\lambda_A} - \bar{\lambda}_{\dot{A}}\frac{\partial}{\partial\bar{\lambda}_{\dot{A}}} + \chi\frac{\partial}{\partial\chi} - s\right)\Phi(\lambda,\chi) = 0\,, \tag{5.7}$$

where $s$ is an integer constant which appears because of a quantum ordering ambiguity in (5.6) and takes several fixed values [24, 25]. As a result, when $a = 0$ the spectrum is restricted to the one of a massless (anti)chiral $N = 1$, $D = 4$ supermultiplet with $s$ being the corresponding superhelicity. On the contrary, when $a \neq 0$ the helicity constraint is absent and one has the infinite tower of massless fields of arbitrary integer and half integer helicity which we shall now show to obey the Vasiliev unfolded higher spin field equations.

For this we should pass from the twistor wave functions (5.4) to $D = 4$ coordinate wave functions which satisfy standard equations of motion of corresponding massless spin fields. This is achieved by performing a Fourier transformation using a prescription analogous to one proposed in [24, 25] (see also [26, 27] for relevant formulations in terms of Lorentz harmonics and 'index' spinors). Before presenting a general formula, let us consider several simple examples.

**s=0.** The spin zero quantum state of the tensorial superparticle is described by the $n = 0$ component $\phi_0(\lambda_A \bar\lambda_{\dot A})$ of (5.4). To get its coordinate representation we just multiply $\phi_0(\lambda_A \bar\lambda_{\dot A})$ by $\exp(ix^m P_m)$ (where $P_m$ is defined in (5.5)) and integrate over $\lambda_A$ and $\bar\lambda_{\dot A}$

$$\phi(x) = \frac{1}{2\pi} \int d^2\lambda\, d^2\bar\lambda\, e^{ix^m P_m}\, \phi_0(\lambda_A \bar\lambda_{\dot A})\,, \qquad P_m = 2\lambda\sigma_m\bar\lambda\,. \tag{5.8}$$

Note that integration involves four independent components of $\lambda_A$ and $\bar\lambda_{\dot A}$ three of which are associated with the components of the lightlike momentum $P_m$ and the fourth one is the phase

$$\lambda_A = e^{i\varphi}\lambda_{0A}\,, \qquad 0 \leq \varphi < 2\pi\,, \tag{5.9}$$

under which $P_m$ of eq. (5.5) is invariant. So the integration over $\varphi$ gives a factor of $2\pi$ which is canceled in (5.8) by the normalization constant. Because of the relation (5.5) the wave function (5.8) satisfies the Klein-Gordon equation

$$\frac{\partial^2}{\partial x^m \partial x_m}\phi(x) = 0\,. \tag{5.10}$$

This equation just reflects the fact that in the classical model there is a first class constraint $P_m P^m = 0$ (2.7) which ensures the quantum states of the superparticle to be massless.

**s=1/2.** The spin $s = \frac{1}{2}$ states come from the $n = 1$ component of $\phi$ or from the $n = 0$ component of $\psi$ (eq. (5.4)). In the latter case the spin 1/2 state is a superpartner of the scalar state discussed above, since $\chi = \theta^A \lambda_A$. Consider, for instance, the function $\bar\lambda_{\dot A}\, \bar\phi^{\dot A}(\lambda\bar\lambda)$. To convert it into the coordinate spinorial wave function we multiply it by $\exp(ix^m P_m)$ and $\lambda_A$ (so that again $\lambda$ and $\bar\lambda$ appear in pairs) and perform the integration as in eq. (5.8)

$$\psi_A(x) = \frac{1}{2\pi} \int d^2\lambda\, d^2\bar\lambda\, e^{ix^m P_m}\, \lambda_A\, \bar\lambda_{\dot A}\, \bar\phi^{\dot A}(\lambda\bar\lambda)\,. \tag{5.11}$$

By construction, in view of (5.5), the spinorial function (5.11) satisfies the Weyl equation for a massless spin 1/2 field

$$\sigma^m_{A\dot A}\, \partial_m\, \psi^A(x) = 0\,. \tag{5.12}$$



JHEP04(2003)013

Note that if we did not multiply $\bar{\lambda}_{\dot{A}}\,\bar{\phi}^{\dot{A}}(\lambda\bar{\lambda})$ by $\lambda_A$ the integrated expression would depend on the phase factor (5.9) and the integral $\int d\varphi\, e^{i\varphi}$ would vanish. This is a generic situation. The integral over $\lambda$ and $\bar{\lambda}$ vanishes unless the number of $\lambda$ and $\bar{\lambda}$ in the integrated expression match.

At $a = 0$ the chiral $N = 1$, $D = 4$ superfield which describes a scalar $N = 1$ supermultiplet of spin 0 and spin 1/2 states is extracted from (5.4) as follows

$$\Phi(x,\theta) = \frac{1}{2\pi}\int d^2\lambda\, d^2\bar{\lambda}\, e^{i(x^m - i\theta\sigma^m\bar{\theta})P_m}\ \Phi(\lambda,\,\bar{\lambda}\,,\chi)\,, \qquad \chi = \lambda_A\,\theta^A\,. \tag{5.13}$$

**s=1.** Using the same procedure as above one gets from (5.4) the self-dual and anti-self-dual field strengths of an abelian gauge field

$$\begin{aligned} F_{AB}(x) &= \frac{1}{2\pi}\int d^2\lambda\, d^2\bar{\lambda}\, e^{ix^mP_m}\ \lambda_A\lambda_B\,\Phi(\lambda,\,\bar{\lambda}\,,\chi)|_{\chi=0} \\ &= \frac{1}{2\pi}\int d^2\lambda\, d^2\bar{\lambda}\, e^{ix^mP_m}\ \lambda_A\lambda_B\,\bar{\lambda}_{\dot{A}_1}\bar{\lambda}_{\dot{A}_2}\,\bar{\phi}^{\dot{A}_1\dot{A}_2}\,, \end{aligned} \tag{5.14}$$

$$\begin{aligned} F_{\dot{A}\dot{B}}(x) &= \frac{1}{2\pi}\int d^2\lambda\, d^2\bar{\lambda}\, e^{ix^mP_m}\ \bar{\lambda}_{\dot{A}}\bar{\lambda}_{\dot{B}}\,\Phi(\lambda,\,\bar{\lambda}\,,\chi)|_{\chi=0} \\ &= \frac{1}{2\pi}\int d^2\lambda\, d^2\bar{\lambda}\, e^{ix^mP_m}\ \bar{\lambda}_{\dot{A}}\bar{\lambda}_{\dot{B}}\,\lambda_{A_1}\lambda_{A_2}\,\phi^{A_1A_2}\,, \end{aligned} \tag{5.15}$$

$F_{AB}(x)$ and $F_{\dot{A}\dot{B}}(x)$ satisfy the Bianchi identities and the Maxwell equations written in the following form

$$\sigma^m_{\dot{A}A}\partial_m\, F^{AB}(x) = 0\,, \qquad \sigma^m_{A\dot{A}}\partial_m\, F^{\dot{A}\dot{B}}(x) = 0\,. \tag{5.16}$$

### 5.1 Higher spins and Vasiliev's unfolded dynamics

We are now in a position to write a general expression which relates the wave function (5.4) with a generating function $C(\bar{a},b,\chi|x)$ which satisfies conformal field equations describing an 'unfolded' higher spin dynamics in flat $D = 4$ space-time [10]–[12]. $C(\bar{a},b,\chi|x)$ depends on auxiliary spinor variables $\bar{a}^{\dot{A}}$, $b^A$ and $\chi$ satisfying star product commutation relations (actually, $\bar{a}^{\dot{A}}$ and $b^A$ are creation operators acting in a Fock space of higher spin states [11])

$$[a_A,\, b^B] = \delta_A^{\ B}\,, \qquad [\bar{a}_{\dot{A}},\, \bar{b}^{\dot{B}}] = \delta_{\dot{A}}^{\ \dot{B}}\,, \qquad \{\chi\,,\bar{\chi}\} = 1 \tag{5.17}$$

and is expressed in terms of $\Phi(\lambda,\bar{\lambda},\chi)$ (eq. (5.4)) as follows

$$C(\bar{a},b,\chi|x) \equiv \sum_{m,n=0}^{\infty}\sum_{k=0}^{1}\frac{1}{m!n!}\bar{a}^{\dot{A}_1}\cdots\bar{a}^{\dot{A}_m}\,b^{A_1}\cdots b^{A_n}\,(\chi)^k\,C_{A_1\cdots A_n\dot{A}_1\cdots\dot{A}_m,\,k}\,(x) \tag{5.18}$$

$$= \sum_{m,n=0}^{\infty}\frac{1}{2\pi m!n!}\int d^2\lambda\, d^2\bar{\lambda}\ (\bar{a}^{\dot{A}}\bar{\lambda}_{\dot{A}})^m\,(b^A\lambda_A)^n\,e^{ix^k\lambda\sigma_k\bar{\lambda}}\,\Phi(\lambda,\bar{\lambda},\chi)\,. \tag{5.19}$$

By construction (5.19) satisfies the equations of unfolded higher spin dynamics (see [10, 11] for details)

$$\frac{\partial^2}{\partial b^A\,\partial\bar{a}^{\dot{A}}}C(\bar{a},b,\chi|x) = \sigma^m_{A\dot{A}}\frac{\partial}{\partial x^m}C(\bar{a},b,\chi|x)\,. \tag{5.20}$$



JHEP04(2003)013

The equation (5.20) reproduces dynamical equations of motion of massless higher spin fields described by holomorphic $C(0,b,\chi|x)$ and anti-holomorphic $C(\bar{a},0,\chi|x)$ components of (5.19) and expresses non-holomorphic components, which are auxiliary fields, as space-time derivatives of the (anti)holomorphic physical fields.

As has been discussed in detail in [11] the unfolded system of equations (5.20) is invariant under an infinite dimensional higher spin algebra $hu(1,1|8)$ which mixes different higher spin components of the generating function (5.19). $hu(1,1|8)$ contains a finite dimensional superconformal subalgebra $osp(2|8)$. This $osp(2|8)$ superconformal symmetry of the higher spin field equations is a manifestation of the $osp(2|8)$ invariance of the classical action (4.1) whose quantization produced the massless higher spin states.

We have thus found a direct relationship of the quantized superparticle in the generalized tensorial coordinate superspace with the unfolded classical dynamics of free higher spin fields by Vasiliev.

## 6. Tensorial superparticles and $AdS$ superspaces. $\mathrm{OSp}(1|2n)$ Cartan forms

In the previous sections we have discussed the dynamics of a superparticle in a generalized superspace parametrized by bosonic tensorial coordinates $X^{\alpha\beta}$ and Grassmann-odd spinor coordinates $\theta^\alpha$ having a flat $N=1$, $D=4$ superspace ($x^m=\gamma^m_{\alpha\beta}X^{\alpha\beta}$, $\theta^\alpha$) as a subspace.

In order to obtain higher spin fields in an $AdS_4$ background we should quantize the superparticle propagating on a tensorial extension of the $AdS_4$ superspace. The reason for this is the following. As we have seen in the previous section, the quantization of the $\mathrm{OSp}(N|8)$ ($N=1,2$) invariant superparticle in the flat tensorial superspace leads to the equations describing free massless higher spin fields. The presence of the tensorial coordinates is crucial for the possibility of obtaining the infinite tower of the higher spin states. For the description of higher spin fields on $AdS_4$ we should therefore find the corresponding generalization of the flat tensorial space to a space which along with the coordinates corresponding to $AdS_4$ contains additional tensorial coordinates.

We shall now analyze whether there exists a generalized superspace which contains in some form an $AdS_4$ superspace $\frac{\mathrm{OSp}(1|4)}{\mathrm{SO}(1,3)}$. An assumption has been made [21] (see also [11]) that this should be a supergroup manifold $\mathrm{OSp}(1|4)$ whose appropriate truncation is known to result in the flat tensorial superspace. Note that both the $N=1$, $D=4$ super Poincaré group extended with the tensorial charge (2.6) and the supergroup $\mathrm{OSp}(1|4)$ are subgroups of $\mathrm{OSp}(1|8)$ and $\mathrm{OSp}(2|8)$ which are symmetries of the tensorial superparticle action (2.1) and (4.1), respectively.

We start the analysis by observing that the generators of the $AdS_4$ boosts can be singled out from the generators of the four dimensional conformal group $\mathrm{SO}(2,4)$ by taking a linear combination of the generators of Poincaré translations $P_m$ and conformal boosts $K_m$, namely $\mathcal{P}_m=P_m+K_m$. The $AdS_4$ manifold can be considered as a coset space $\frac{\mathrm{SO}(2,4)}{(\mathrm{SO}(1,3)\times D)\otimes K}$ parametrized by the coset element $e^{\mathcal{P}_m x^m}$.

Analogously, for the case of tensorial extension of $AdS_4$ space let us consider the generators

$$\mathcal{P}_{\alpha\beta}=P_{\alpha\beta}+K_{\alpha\beta}\,,\qquad [\mathcal{P},\mathcal{P}]\sim M\,,\qquad [\mathcal{P},M]\sim\mathcal{P}\,, \tag{6.1}$$

where $M_{\alpha\beta}$ stand for the generators of Sp(4) (3.11). One can see that $\mathcal{P}_{\alpha\beta}$ and $M_{\alpha\beta}$ form the algebra of $\mathrm{Sp}_L(4) \times \mathrm{Sp}_R(4)$, where Sp(4) generated by $M_{\alpha\beta}$ is the diagonal subgroup. The generators of $\mathrm{Sp}_L(4)$ are $M_L = P + K + M$ and that of $\mathrm{Sp}_R(4)$ are $M_R = P + K - M$. Thus the extended tensorial $AdS_4$ is a coset (group) manifold

$$\mathrm{Sp}(4) \sim \frac{\mathrm{Sp}_L(4) \times \mathrm{Sp}_R(4)}{\mathrm{Sp}(4)} . \tag{6.2}$$

Sp(4) is a diagonal subgroup of $\mathrm{Sp}_L(4) \times \mathrm{Sp}_R(4)$, which is a subgroup of Sp(8) and this manifold can be realized as a coset space $\frac{\mathrm{Sp}(8)}{\mathrm{GL}(4) \otimes K}$ with the coset element $e^{(P+K)_{\alpha\beta} X^{\alpha\beta}}$.

Let us note also that, as we have discussed in section 3, the flat tensorial space associated with $P_{\alpha\beta}$ can be realized as a formally similar but different coset space $\frac{\mathrm{Sp}(8)}{\mathrm{GL}(4) \otimes K}$ with $e^{P_{\alpha\beta} X^{\alpha\beta}}$ being the coset element.

As to the superspace generalization, to get the tensorial extension of super $AdS_4$ we should add to the bosonic generators $\mathcal{P} = P + K$ the fermionic generators

$$\mathcal{Q} = Q + S , \qquad \{\mathcal{Q}, \mathcal{Q}\} \sim P + K + M = M_L . \tag{6.3}$$

One can see that $\mathcal{Q}$ and $M_L = P + K + M$ form the superalgebra of $\mathrm{OSp}_L(1|4)$, and the coset superspace in question is

$$\frac{\mathrm{OSp}_L(1|4) \times \mathrm{Sp}_R(4)}{\mathrm{Sp}(4)} . \tag{6.4}$$

One should also note that this supercoset is isomorphic to OSp(1|4). I.e. the simplest tensorial extension of the $N = 1$ supersymmetric $AdS_4$ is the supergroup manifold OSp(1|4). Like the flat tensorial superspace (see the discussion after eq. (3.18)) the tensorial extension of the $AdS_4$ superspace can be realized as a coset superspace

$$\frac{\mathrm{OSp}(1|8)}{\mathrm{GL}(4) \otimes SK} , \tag{6.5}$$

both being embedded in a certain way into OSp(1|8).

The next question we would like to address is whether the supergroup manifold OSp(1|4) is related via a GL(4) transformation to the flat tensorial superspace whose supervielbeins transform under superconformal Sp(1|8) by induced GL(4) rotations as has been shown in eqs. (3.17)–(3.19). We know from [23] that the conventional $AdS_4$ superspace $\frac{\mathrm{OSp}(1|4)}{\mathrm{SO}(1,3)}$ is superconformally flat, i.e. the corresponding supervielbeins can be written in the form

$$E^{A\dot{A}} = e^{\Phi(x,\theta,\bar{\theta})} \Pi^{A\dot{A}} = e^{\Phi(x,\theta,\bar{\theta})} \left( dX^{A\dot{A}} - id\theta^A \bar{\theta}^{\dot{A}} + i\theta^A d\bar{\theta}^{\dot{A}} \right) \tag{6.6}$$

$$E^A = e^{\Phi(x,\theta,\bar{\theta}) + iW(x,\theta,\bar{\theta})} \left( d\theta^A + 2i\Pi^{A\dot{A}} \bar{D}_{\dot{A}} \Phi \right) , \tag{6.7}$$

where $\Phi(x, \theta, \bar{\theta})$ and $W(x, \theta, \bar{\theta})$ are superfunctions. We see, for example, that the vector $AdS_4$ supervielbein (6.6) is related to the flat superspace supervielbein $\Pi^{A\dot{A}}$ by a Weyl rescaling. For an earlier detailed discussion of various aspects of superconformal field theories in the $AdS_4$ superspace and their relation to superconformal theories in flat superspace see [28].

In our case of the tensorial superspaces the role of the "Weyl" factor is played by the GL(4) matrices (see (3.19)). An analogy with (3.19) allows us to assume that the bosonic supervielbeins $\Omega^{\alpha\beta}$ of OSp(1|4) can be put into the form similar to (3.19) and in this sense are 'GL(4) flat'. Actually, this turns out to be the case for an arbitrary supergroup manifold OSp(1|2$n$) as we shall show below.

Let us first consider the case of a bosonic simplectic group manifold Sp(2$n$) ($n = 1,\ldots,\infty$). We call a manifold 'GL(2$n$) flat' if its vielbein (or Cartan form) $\omega^{\alpha\beta} = dX^{\mu\nu}\,\omega_{\mu\nu}{}^{\alpha\,\beta}(X)$ is flat up to a GL(2$n$) rotation[1]

$$\omega^{\alpha\beta}(X) = dX^{\alpha'\beta'}\; G_{\alpha'}{}^{\alpha}\,(X) G_{\beta'}{}^{\beta}\,(X)\,. \tag{6.8}$$

The Maurer-Cartan equations for the group manifold Sp(2$n$) have the form

$$d\omega^{\alpha\beta} + \frac{\alpha}{2}\omega^{\alpha\gamma}\wedge\omega_{\gamma}{}^{\beta} = 0\,. \tag{6.9}$$

Substituting (6.8) into (6.9) and solving for $G_{\alpha'}{}^{\alpha}$ one gets

$$(G_{\alpha'}{}^{\alpha})^{-1} = \delta_{\alpha'}{}^{\alpha} + \frac{\alpha}{4}X_{\alpha'}{}^{\alpha}, \quad G_{\alpha'}{}^{\alpha} = \delta_{\alpha'}{}^{\alpha} + \sum_{n=1}^{\infty}\left(-\frac{\alpha}{4}\right)^{n}\,(X)^{n}{}_{\alpha'}{}^{\alpha}\,, \tag{6.10}$$

where $\alpha$ is a parameter of inverse length dimension, which in the case of Sp(4) is proportional to the inverse $AdS_4$ radius $\alpha = \frac{4}{R}$. Thus the Sp(2$n$) is GL(2$n$) flat and it reduces to the flat tensorial space of sections 2–5 when $\alpha = 0$.

These results are generalized to a super group manifold OSp(1|2$n$) whose Maurer-Cartan equations have the form

$$d\Omega^{\alpha\beta} + \frac{\alpha}{2}\Omega^{\alpha\gamma}\wedge\Omega_{\gamma}{}^{\beta} \;=\; -iE^{\alpha}\wedge E^{\beta} \tag{6.11}$$

$$dE^{\alpha} + \frac{\alpha}{2}E^{\beta}\wedge\Omega_{\beta}{}^{\alpha} \;=\; 0\,. \tag{6.12}$$

After introducing the new Grassmann variable $\Theta^{\alpha}$

$$\theta^{\alpha} = \Theta^{\alpha'}(G_{\alpha'}{}^{\alpha})^{-1}P^{-1}(\Theta^2)\,, \tag{6.13}$$

one can verify that the Maurer-Cartan equations (6.11) are satisfied by the following GL(2$n$) flat ansatz

$$\Omega^{\alpha\beta} \;=\; \Pi^{\alpha'\beta'}\,\mathcal{G}_{\alpha'}{}^{\alpha}\mathcal{G}_{\beta'}{}^{\beta}\,, \qquad \mathcal{G}_{\alpha'}{}^{\alpha} = G_{\alpha'}{}^{\alpha}\,(X) - \frac{i\alpha}{8}(\Theta_{\alpha'} - 2G_{\alpha'}{}^{\gamma}\Theta_{\gamma})\Theta^{\alpha}\,, \tag{6.14}$$

$$E^{\alpha} \;=\; P(\Theta^2)\mathcal{D}\Theta^{\alpha} - \Theta^{\alpha}\mathcal{D}P(\Theta^2), \quad P(\Theta^2) = \sqrt{1 + \frac{i\alpha}{8}\Theta^{\beta}\Theta_{\beta}}\,, \tag{6.15}$$

[1] We should stress that this is a nontrivial property in that it implies that the Cartan form component matrix $\omega_{\mu\nu}{}^{\alpha\,\beta}(X)$ is a "direct product" of GL(2$n$) matrix components $\omega_{\mu\nu}{}^{\alpha\,\beta}(X) = \frac{1}{2}[G_{\mu}{}^{\alpha}\,(X)G_{\nu}{}^{\beta}\,(X) + G_{\mu}{}^{\beta}\,(X)G_{\nu}{}^{\alpha}\,(X)]$, which of course does not take place in the case of a generic matrix.



JHEP04(2003)013

where $\mathcal{D} = \delta_\alpha^\beta\, d + \frac{\alpha}{2}\omega_\alpha{}^\beta(X)$ is the covariant derivative, $G_{\alpha'}{}^\alpha(X)$ has been defined in (6.10) and $\Pi^{\alpha\beta} = dX^{\alpha\beta} - \frac{i}{2}(d\theta^\alpha\theta^\beta + d\theta^\beta\theta^\alpha)$ being the flat superform (3.17). The bosonic Cartan form (6.15) is bilinear in the Grassmann coordinates $\Theta$ (6.13), as has been found in [21]

$$\Omega^{\alpha\beta} = \omega^{\alpha\beta}(X) + \frac{i}{2}(\Theta^\alpha \mathcal{D}\Theta^\beta + \Theta^\beta \mathcal{D}\Theta^\alpha)\,, \tag{6.16}$$

which is an indirect check of the GL($2n$) flat ansatz.

## 7. Quantization of the superparticle on OSp(1|4) and higher spin fields on the $AdS_4$ superbackground

As it could be expected from the discussion of the previous section the quantization of the superparticle propagating on the OSp(1|4) supergroup manifold leads to the theory of massless higher spin fields in the $AdS_4$ background.

In order to show this explicitly let us start with the OSp(1|4) invariant superparticle action of [21]

$$S = \int d\tau \Lambda_\alpha \Lambda_\beta\, \Omega^{\alpha\beta}\,, \tag{7.1}$$

where $\Omega^{\alpha\beta}$ has been written in (6.16) and $\Lambda^\alpha$ is a commuting Majorana spinor similar to $\lambda^\alpha$.

As it has been proved in the previous section the OSp(1|4) supergroup manifold (being the tensorial extension of the $AdS_4$ superspace) is GL(4) flat, i.e. its Grassmann-even Cartan form can be presented in the form (6.14). This property essentially simplifies the analysis and the quantization of the action (7.1) since after the redefinition of the twistor variables

$$\Lambda_\alpha = \mathcal{G}^{-1\,\alpha'}_{\alpha}(X,\theta)\lambda_{\alpha'}$$

the action takes the 'flat' form

$$S = \int d\tau \lambda_\alpha \lambda_\beta \Pi^{\alpha\beta} \tag{7.2}$$

and can be further rewritten in the pure supertwistor form (2.12)–(2.14). Therefore the OSp(1|4) model is classically equivalent to the superparticle in the flat tensorial superspace. The group theoretical reason for this is that both superspaces are realized as a coset superspace $\frac{\mathrm{OSp}(1|8)}{\mathrm{GL}(4)\otimes SK}$. As a consequence, when quantizing the system on OSp(1|4) one can follow the same lines as in the flat case. Namely, due to the classical equivalence between the flat and OSp(1|4) superparticle the quantum states of the latter are again described by the 'twistorial' wave functions (5.4), but their 'Fourier' transform to the coordinate wave functions on the $AdS_4$ superspace should be performed in a different way. These wave functions should now obey unfolded equations for higher spin fields on $AdS_4$. In other words performing quantization and deriving the equations of motion one should respect the symmetries of the original classical superparticle model on the tensorial extension of $AdS_4$ (see [23] for the relevant discussion).

As an illustration consider first several examples corresponding to the bosonic $AdS_4$ case. Let us take the $AdS_4$ metric in a conformally flat form

$$g_{mn} = e^{\rho(x)}\eta_{mn}\,, \tag{7.3}$$

where[2] $\rho(x) = \ln \frac{4}{(1-x^2)^2}$, and the corresponding vielbeins and the spin connection are

$$\begin{aligned} e^a_m &= e^{\frac{\rho(x)}{2}} \delta^a_m \,, \qquad e^m_a = e^{-\frac{\rho(x)}{2}} \delta^m_a \,, \\ \omega_{m,ab} &= \frac{1}{2}(\eta_{am}\partial_b\rho(x) - \eta_{bm}\partial_a\rho(x)) \,, \qquad \omega_{c,ab} = e^m_c \omega_{m,ab} = e^{-\frac{\rho(x)}{2}} \omega_{c,ab} \,. \end{aligned} \tag{7.4}$$

For our purposes it is useful to rewrite these expressions in the two component Weyl spinor notation:

$$e^{A\dot{A}}_{M\dot{M}} = e^{\frac{\rho(x)}{2}} \delta^A_M \delta^{\dot{A}}_{\dot{M}}, \quad \omega_{A\dot{A},BC} = -\frac{1}{4} e^{-\frac{\rho(x)}{2}} (\epsilon_{BA}\partial_{\dot{A}C}\rho(x) + \epsilon_{CA}\partial_{\dot{A}B}\rho(x)) \,, \tag{7.5}$$

then the $AdS_4$ covariant derivative is defined as

$$D_{M\dot{M}}\Phi_{A\dot{A}} = -\frac{1}{2}\sigma^m_{M\dot{M}}\partial_m\Phi_{A\dot{A}} + \omega_{M\dot{M},A}{}^{B}\Phi_{B\dot{A}} + \omega_{M\dot{M},\dot{A}}{}^{\dot{B}}\Phi_{A\dot{B}} \,. \tag{7.6}$$

**s=0.** By the analogy with section 5 we take the following ansatz for the scalar field wave function on $AdS_4$

$$\phi(x) = \frac{1}{2\pi} \int d^2\lambda\, d^2\bar{\lambda}\, e^{ix^m P_m - \frac{1}{2}\rho(x)}\, \phi_0(\lambda_A\bar{\lambda}_{\dot{A}}) \,, \tag{7.7}$$

which satisfies the following field equation

$$(D_m D^m + 2)\phi(x) = 0 \,, \tag{7.8}$$

i.e. the conformally invariant equation for a massless scalar field propagating on the four dimensional anti de Sitter space [6]–[8] and [23], $D_m = \partial_m + \omega_m$ being the $AdS_4$ covariant derivative.

The form of the equation (7.8) is in a full correspondence with our previous discussion. When considering the quantum version of the classical constraint (2.7) the momentum operator $P_m = -i\frac{\partial}{\partial x^m}$ becomes non hermitean in the anti de Sitter background and one should consider instead the covariant laplacian, while because of the quantum ordering ambiguity, the value of the "effective mass" appearing in (7.8) can be fixed by the requirement of the conformal invariance of the mass shell equation [6].

**s=1/2.** We take the ansatz for the wave function in the form

$$\psi_A(x) = \frac{1}{2\pi} \int d^2\lambda\, d^2\bar{\lambda}\, e^{ix^m P_m - \frac{3}{4}\rho(x)}\, \lambda_A\, \bar{\lambda}_{\dot{A}}\, \bar{\phi}^{\dot{A}}(\lambda\bar{\lambda}) \,, \quad P_m = 2\lambda\sigma_m\bar{\lambda} \,. \tag{7.9}$$

which is an $AdS_4$ generalization of the corresponding flat wave function (5.11). The value $-3/4$ of the $\rho(x)$ factor being now fixed by the requirement for (7.9) to satisfy the $AdS_4$ Dirac equation

$$e^{M\dot{M}}_{A\dot{A}}\, D_{M\dot{M}}\, \psi^A(x) = 0 \,. \tag{7.10}$$

The procedure described for the spin 1/2 field is straightforwardly generalized to fields of arbitrary (half)integer spin $s = n/2$. The wave function of the form

$$\begin{aligned} &\psi_{A_1,\ldots,A_{2s}}(x) = \\ &\quad = \frac{1}{2\pi} \int d^2\lambda\, d^2\bar{\lambda}\, e^{ix^{A\dot{A}}\lambda_A\bar{\lambda}_{\dot{A}} - \frac{1+s}{2}\rho(x)}\, \lambda_{A_1} \ldots \lambda_{A_{2s}} \bar{\lambda}_{\dot{A}_1} \ldots, \bar{\lambda}_{\dot{A}_{2s}}\, \bar{\phi}^{\dot{A}\cdots\dot{A}_{2s}}(\lambda\bar{\lambda}) \,, \end{aligned} \tag{7.11}$$

[2]For simplicity we put the radius of the anti de Sitter space equal to one.

satisfies the higher spin field equation

$$e^{M\dot{M}}_{A_1\dot{A}}\, D_{M\dot{M}}\, \psi^{A_1,\ldots,A_{2s}}(x) = 0\,. \tag{7.12}$$

We are now in a position to establish the relationship of the quantized particle on the group manifold Sp(4) with the unfolded dynamics of higher spin fields in $AdS_4$. In the unfolded formulation [1, 10, 11] a field with the spin $s$ is described by an infinite chain of equations

$$\begin{aligned} &D_{M\dot{M}} C_{A_1\ldots A_{n+2s}\,;\,\dot{A}_1\ldots\dot{A}_n}(x) = \\ &\quad = e^{A\dot{A}}_{M\dot{M}} C_{A_1,\ldots A_{n+2s},A;\dot{A}_1\ldots\dot{A}_n,\dot{A}}(x) - n(n+2s) e_{M\dot{M},\{A_1\dot{A}_1} C_{A_2\ldots A_{n+2s};\dot{A}_2\ldots\dot{A}_n\}}(x) \end{aligned} \tag{7.13}$$

and by their complex conjugate. The scalar field is described by the set of fields with an equal number of dotted and undotted indexes. The fields $C_{A_1\ldots A_{2s}\,;\,0}(x)$, $C_{0;\,\dot{A}_1\ldots A_{2s}}(x)$ and $C_{0;0}$ (for the scalar field) are physical and all the other are auxiliary. For spin $s \neq 0$ one obtains from the first equation in the chain (7.13) the dynamical equation of the physical field

$$e^{M\dot{M}}_{A_1\dot{A}}\, D_{M\dot{M}}\, C^{A_1\ldots A_{2s}}(x) = 0 \tag{7.14}$$

while the other equations imply no further conditions on the physical field but just express the auxiliary fields via derivatives of the physical field. For the scalar field the situation is slightly different. From the first two equations in the chain (7.13) one obtains the Klein-Gordon equation for the field $C_{0,0}(x)$, while the other equations express the auxiliary fields in terms of the higher covariant derivatives of the basic scalar field.

Now one can explicitly see that by identifying the wave function (7.7) with $C_{0,\,0}(x)$, the wave function (7.11) with $C_{A_1\ldots A_{2s}}(x)$ and $D^n_{\{A\dot{A}} C_{A_1\ldots A_{2s}\}}(x)$ with the auxiliary fields one obtains the complete correspondence between the superparticle model on Sp(4) and the unfolded massless higher spin dynamics in the $AdS_4$ space.

We should note that higher spin plane wave solutions on $AdS_4$ have been obtained in [29] along with the corresponding generating functional

$$C_{\rm plane}(y,\bar{y}|x) \;=\; = c_0 \exp\left\{ i(y_A\bar{y}_{\dot{A}} + \lambda_A\bar{\lambda}_{\dot{A}})x^{A\dot{A}} - \frac{\rho(x)}{2} + (1-x^2)^{\frac{1}{2}}(y^A\lambda_A + \bar{y}^{\dot{A}}\bar{\lambda}_{\dot{A}}) \right\}, \tag{7.15}$$

$$C^{\rm plane}_{A_1\ldots A_{2s}}(x) \;=\; \frac{\partial}{\partial y^{A_1}} \cdots \frac{\partial}{\partial y^{A_{2s}}}\, C_{\rm plane}(y,\bar{y}|x)|_{y=\bar{y}=0}\,, \tag{7.16}$$

where $c_0$ is a constant and the auxiliary oscillator variables $y^A$ and $\bar{y}^{\dot{A}}$ satisfy the Moyal star product commutation relations [11]

$$[y^A\,, y^B] = 2i\varepsilon^{AB}\,, \qquad [\bar{y}^{\dot{A}}\,, \bar{y}^{\dot{B}}] = 2i\varepsilon^{\dot{A}\dot{B}}\,. \tag{7.17}$$

The generating functional for the generic solution (7.11) of the higher spin equations (7.13) on $AdS_4$ is obtained by replacing $c_0$ in (7.15) with

$$\Phi(\lambda_A,\,\bar{\lambda}_{\dot{A}}) = \sum_{n=0}^{\infty} \left[ \lambda_{A_1}\cdots\lambda_{A_n}\,\phi^{A_1\cdots A_n}(\lambda_A\bar{\lambda}_{\dot{A}}) + \bar{\lambda}_{\dot{A}_1}\cdots\bar{\lambda}_{\dot{A}_n}\bar{\phi}^{\dot{A}_1\cdots\dot{A}_n}(\lambda_A\bar{\lambda}_{\dot{A}}) \right],$$

and integrating over $\lambda_A$ and $\bar{\lambda}_{\dot{A}}$

$$C(y,\bar{y}|\,x) = \tag{7.18}$$
$$= \frac{1}{2\pi}\int d^2\lambda\, d^2\bar{\lambda}\, \Phi(\lambda,\bar{\lambda})\, exp\left\{ i(y_A\bar{y}_{\dot{A}} + \lambda_A\bar{\lambda}_{\dot{A}})x^{A\dot{A}} - \frac{\rho(x)}{2} + (1-x^2)^{\frac{1}{2}}(y^A\lambda_A + \bar{y}^{\dot{A}}\bar{\lambda}_{\dot{A}}) \right\}.$$

The connection between the unfolded formulation of higher spin fields in $AdS_4$ and the quantum spectrum of the particle on the group manifold Sp(4) established above can be generalized to the supersymmetric case. The generating functional (7.18) will now also depend on the Grassmann-odd supertwistor variable $\chi$. For example, this can be done in the framework of a minimal $N = 1$ higher spin field theory which corresponds to the OSp(1|4) supersymmetry [13, 14]. In this formulation the bosonic part of the generating functional contains fields with only even values of spin, while the wave functions of the superpartners are related to each other by acting on the bosonic (fermionic) part of the generating functional $C(y,\bar{y}|x)$ with the supersymmetry generator $\mathcal{Q}$ (6.3) which in the unfolded formulation is realized as $\mathcal{Q}_\alpha = y_\alpha(\chi+\bar{\chi})$ (see sections 3 and 6). Thus according to [13, 14] one can obtain the supersymmetric description of the fields (7.11) grouped into irreducible $AdS_4$ supermultiplets.

## 8. Conclusion

We have demonstrated that the quantization of the superparticle propagating in the flat tensorial superspace and on OSp(1|4), which are different cosets $\frac{\mathrm{OSp}(1|8)}{\mathrm{GL}(4)\otimes SK}$, produces the free massless higher spin field theory in flat and $AdS_4$ superspaces, respectively. An important property of OSp(1|4), and in general of OSp(1|2$n$), which we have revealed and which has allowed us to find the explicit spectrum of quantum higher spin states of the superparticle on $AdS_4$ is the GL(2$n$) flatness of these supergroup manifolds. It would be of interest to analyse whether other groups and supergroups possess this property. The connection with the unfolded dynamics of higher spin fields has been achieved for both, the flat and $AdS_4$ superbackground by taking the appropriate "Fourier" transformation of the wave function of the quantized superparticle.

The fact that the quantization of the superparticle dynamics on the tensorial superspace $\frac{\mathrm{OSp}(1|8)}{\mathrm{GL}(4)\otimes SK}$ results in the dynamics of massless higher spin fields in an associated 4D superspace-time is in a complete correspondence with the results of [11], where an alternative quantization procedure was applied, and with [12] where it has been shown that the requirement of the causality and locality of the theory on the tensorial space singles out the 4D space-time as a subspace of the tensorial space on which the dynamical higher spin fields are localized.

The natural development of these results would be to consider the dynamics of the superparticle in higher dimensional and curved superbackgrounds and to study the possibility of introducing interactions of the higher spin fields in this way.

Let us also note that the explicit GL(2$n$) flat representation of the Cartan forms of the supergroups OSp(1|2$n$) found in section 6 can be useful for many applications where these supergroups appear, for instance, for the analysis of a conjectured OSp(1|32) and OSp(1|64) structure of M-theory (see [23] for references).



JHEP04(2003)013

## Acknowledgments

The authors are grateful to I. Bandos, E. Ivanov, P. Pasti, M. Tonin and M. Vasiliev for interest to this work and useful discussions. This work was partially supported by the Grants 1010073 and 7010073 from FONDECYT (Chile) (M.P. and D.S.), by the Grant N 383 of the Ukrainian State Fund for Fundamental Research (D.S.), by the INTAS Research Project N. 2000-254 and by the European Community's Human Potential Programme under contract HPRN-CT-2000-00131 Quantum Spacetime (D.S. and M.T.).

**Note added.** When this article was ready for publication, the paper [30] appeared on the net in which, in particular, Cartan forms and solutions of the free massless field equations in the tensorial AdS (super)space, which generalize (7.15), have been constructed using a star product realization of the $osp\,(N|2n)$ superalgebra. One can assume that the construction of [30] should essentially simplify with the use of the $\mathrm{GL}(2n)$ flatness of the $\mathrm{OSp}(1|2n)$ Cartan forms found in the present paper.

## References

[1] M.A. Vasiliev, *Higher spin gauge theories: star-product and AdS space*, hep-th/9910096; *Higher spin symmetries, star-product and relativistic equations in AdS space*, hep-th/0002183; *Progress in higher spin gauge theories*, hep-th/0104246.

[2] B. Sundborg, *Stringy gravity, interacting tensionless strings and massless higher spins*, *Nucl. Phys.* **102** *(Proc. Suppl.)* (2001) 113 [hep-th/0103247].

[3] L. Dolan, C.R. Nappi and E. Witten, *Conformal operators for partially massless states*, *J. High Energy Phys.* **10** (2001) 016 [hep-th/0109096].

[4] E.S. Fradkin and M.A. Vasiliev, *On the gravitational interaction of massless higher spin fields*, *Phys. Lett.* **B 189** (1987) 89; *Cubic interaction in extended theories of massless higher spin fields*, *Nucl. Phys.* **B 291** (1987) 141.

[5] A.Y. Segal, *Conformal higher spin theory*, hep-th/0207212.

[6] C. Fronsdal, *Singletons and massless, integral spin fields on de Sitter space (elementary particles in a curved space VII)*, *Phys. Rev.* **D 20** (1979) 848.

[7] R.R. Metsaev, *Massless mixed symmetry bosonic free fields in D-dimensional anti-de Sitter space-time*, *Phys. Lett.* **B 354** (1995) 295; *Arbitrary spin massless bosonic fields in d-dimensional anti-de Sitter space*, hep-th/9810231.

[8] I.L. Buchbinder, A. Pashnev and M. Tsulaia, *Lagrangian formulation of the massless higher integer spin fields in the AdS background*, *Phys. Lett.* **B 523** (2001) 338 [hep-th/0109067]; *Massless higher spin fields in the AdS background and brst constructions for nonlinear algebras*, hep-th/0206026.

[9] D. Francia and A. Sagnotti, *Free geometric equations for higher spins*, *Phys. Lett.* **B 543** (2002) 303 [hep-th/0207002]; *On the geometry of higher-spin gauge fields*, hep-th/0212185.

[10] M.A. Vasiliev, *Equations of motion of interacting massless fields of all spins as a free differential algebra*, *Phys. Lett.* **B 209** (1988) 491; *Consistent equations for interacting massless fields of all spins in the first order in curvatures*, *Ann. Phys. (NY)* **190** (1989) 59.

[11] M.A. Vasiliev, *Conformal higher spin symmetries of 4d massless supermultiplets and* $\mathrm{OSp}(L,2M)$ *invariant equations in generalized (super)space*, *Phys. Rev.* **D 66** (2002) 066006 [hep-th/0106149].

[12] M.A. Vasiliev, *Relativity, causality, locality, quantization and duality in the* $\mathrm{Sp}(2M)$ *invariant generalized space-time*, hep-th/0111119; *Cubic interactions of bosonic higher spin gauge fields in* $AdS_5$, *Nucl. Phys.* **B 616** (2001) 106 [hep-th/0106200].

[13] J. Engquist, E. Sezgin and P. Sundell, *On* $N = 1, 2, 4$ *higher spin gauge theories in four dimensions*, *Class. and Quant. Grav.* **19** (2002) 6175 [hep-th/0207101].

[14] J. Engquist, E. Sezgin and P. Sundell, *Superspace formulation of 4d higher spin gauge theory*, hep-th/0211113.

[15] E. Sezgin and P. Sundell, *Analysis of higher spin field equations in four dimensions*, *J. High Energy Phys.* **07** (2002) 055 [hep-th/0205132]; *Massless higher spins and holography*, *Nucl. Phys.* **B 644** (2002) 303 [hep-th/0205131]; *7d bosonic higher spin theory: symmetry algebra and linearized constraints*, *Nucl. Phys.* **B 634** (2002) 120 [hep-th/0112100].

[16] I. Bandos and J. Lukierski, *Tensorial central charges and new superparticle models with fundamental spinor coordinates*, *Mod. Phys. Lett.* **A 14** (1999) 1257 [hep-th/9811022].

[17] S. Fedoruk and V.G. Zima, *Massive superparticle with tensorial central charges*, *Mod. Phys. Lett.* **A 15** (2000) 2281 [hep-th/0009166]; *Uniform twistor-like formulation of massive and massless superparticles with tensorial central charges*, *Nucl. Phys.* **102** *(Proc. Suppl.)* (2001) 233 [hep-th/0104178].

[18] I.A. Bandos, J.A. de Azcarraga, J.M. Izquierdo and J. Lukierski, *BPS states in M-theory and twistorial constituents*, *Phys. Rev. Lett.* **86** (2001) 4451 [hep-th/0101113];
I.A. Bandos, *BPS preons and tensionless super-p-branes in generalized superspace*, *Phys. Lett.* **B 558** (2003) 197 [hep-th/0208110].

[19] R. Mkrtchyan, *Brain content of branes' states*, *Phys. Lett.* **B 558** (2003) 205 [hep-th/0212174].

[20] I. Bandos, J. Lukierski and D.P. Sorokin, *Superparticle models with tensorial central charges*, *Phys. Rev.* **D 61** (2000) 045002 [hep-th/9904109]; *The* $\mathrm{OSp}(1|4)$ *superparticle and exotic BPS states*, hep-th/9912264.

[21] I. Bandos, J. Lukierski, C. Preitschopf and D.P. Sorokin, OSp *supergroup manifolds, superparticles and supertwistors*, *Phys. Rev.* **D 61** (2000) 065009 [hep-th/9907113].

[22] C. Fronsdal, *Massless particles, ortosymplectic symmetry and another type of Kaluza-Klein theory*, in *Essays on supersymmetry*, Mathematical Physics Studies **8**, Reidel, 1986.

[23] I. Bandos, E. Ivanov, J. Lukierski and D. Sorokin, *On the superconformal flatness of AdS superspaces*, *J. High Energy Phys.* **06** (2002) 040 [hep-th/0205104];
*The superconformal structure of AdS superspaces*, Preprint DFPD 02/TH/28. To appear in the Proceedings of the 3d International Sakharov Conference, June 2002, Moscow.

[24] Y. Eisenberg, *Supertwistors and superpoincare invariant actions for all linearized extended supersymmetric theories in four-dimensions*, *Mod. Phys. Lett.* **A 4** (1989) 195;
Y. Eisenberg and S. Solomon, *(Super)field theories from (super)twistors*, *Phys. Lett.* **B 220** (1989) 562.

[25] M.S. Plyushchay, *Covariant quantization of massless superparticle in four-dimensional space-time: twistor approach*, *Mod. Phys. Lett.* **A 4** (1989) 1827; *Lagrangian formulation for the massless (super)particles in (super)twistor approach*, *Phys. Lett.* **B 240** (1990) 133.

[26] I.A. Bandos, *Multivalued action functionals, Lorentz harmonics, and spin*, *Sov. Phys. JETP Lett.* **52** (1990) 205; *A superparticle in Lorentz-harmonic superspace*, *Sov. J. Nucl. Phys.* **51** (1990) 906.

[27] V.G. Zima and S.O. Fedoruk, *Spinor (super)particle with a commuting index spinor*, *Sov. Phys. JETP Lett.* **61** (1995) 251 [*Zh. Eksp. Teor. Fiz.* **61** (1995) 241]; *Covariant quantization of $D = 4$ brink-schwarz superparticle with Lorentz harmonics*, *Theor. Math. Phys.* **102** (1995) 305 [hep-th/9409117]; *Weinberg propagator of free massive arbitrary spin particle from bfv-brst path integral*, *Class. and Quant. Grav.* **16** (1999) 3653 [hep-th/9807192].

[28] E.A. Ivanov and A.S. Sorin, *Wess-zumino model as linear sigma model of spontaneously broken conformal and* $\mathrm{OSp}(1,4)$ *supersymmetries*, *Sov. J. Nucl. Phys.* **30** (1979) 440; *Superfield formulation of* $\mathrm{OSp}(1,4)$ *supersymmetry*, *J. Phys.* **A 13** (1980) 1159.

[29] K.I. Bolotin and M.A. Vasiliev, *Star-product and massless free field dynamics in $AdS_4$*, *Phys. Lett.* **B 479** (2000) 421 [hep-th/0001031].

[30] V.E. Didenko and M.A. Vasiliev, *Free field dynamics in the generalized AdS (super)space*, hep-th/0301054.